OSCILLATIONS AND UNDULATIONS IN SHEAR FLOWS © (copy at Lerners LLD, 11/05/17)

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A rigid simple harmonic (SH) undulating boundary is one with a steady shape as in the galvanized steel sheets used by Benjamin in his study of simple harmonic (SH) elements in shear flows along flat boundaries[1]. These undulations represent the paths along which laminae flow. Each flowing lamina conforms to the form of the undulations, flowing up and down with SH periodicity as SH long-crested oscillations. Water is incompressible and cannot decompress without cavitation. Thus, in water flows, each lamina flowing over a SH undulating boundary must have SH wavy paths conforming to the shape of the boundary – as must its neighbours – layer upon layer.

Although each lamina, flowing as a SH long-crested boundary layer shear oscillation, or as an oscillation along a SH wavy boundary, flows at its own velocity, which increases as the distance from the boundary increases; regardless of individual laminar velocities each descends synchronously (in phase), striking towards the boundary and then rebounding in phase, creating SH vibrations and SH sound, just as any oscillation of a mass in a fluid creates a sound wave.

If the corrugated panel were lubricated and resting on a flat surface, aligned with the undulations normal to the flow, the wavy boundary would be entrained, sliding along as a travelling wavy boundary at a velocity considerably less than the average shear-flow velocity. Sliding along under the crests of the undulations there must be SH spaces that move at the speed of the rigid boundary undulations. If lubricating fluid filled these spaces, SH long columnar eddies would develop under the wave crests – similar to the long columnar eddies that develop under the crests of water waves in the wind and which roll over and crash on the shore as long-crested SH “breakers” or “rollers”, as the lower margins of the columnar eddies are slowed by the sloping shallows.

The preformed SH stationary galvanized SH undulations are analogous to:

1. the water/carbon bisulfide interface waves in the Reynolds U-tube standing wave experiment, with SH oscillations in the laminae flowing in opposite directions on each side of the SH shear wave interface undulations (the paths followed by the flowing fluid oscillations)
2. Bagnold’s long-crested SH shear waves developing in sand in wind and water oscillating shear flows
3. Essapian’s SH circumferential (i.e., long-crested) undulations on dolphin epidermis over which water flows as oscillations that strike at the boundary as SH oscillations
4. SH circumferential undulations on the inside of arteries (arteriographic standing waves)
5. Benjamin’s SH undulations on viscous syrup (may be sliding very slowly streamwise)

The preformed galvanized SH sliding (“travelling”) undulations are analogous to:

[1] T. B. Benjamin, Shearing Flow Over a Wavy Boundary J. Fluid. Mech., 6, 161-205 (1959)

1. Travelling (entrained) waves on water (a compliant laminar fluid), over which flows the wind (also a compliant laminar fluid), as oscillations
2. Inter-stratum SH waves in the sky (two strata of compliant laminar air along a fluid shear interface similar to Benjamin's H_2O/CS_2 U-tube liquid interface shear waves), often outlined by SH clouds which reveal the more slowly moving paths travelled by the more rapidly moving air oscillations on each side of the shear inter-stratum interface.
3. Thomas glass bead SH undulations that form in transition, growing in turbulence and which slide along a shiny glass cylinder as travelling waves. Thomas considered the glass bead waves to be similar to Bagnold's sand waves.